Introduction to Simulation and modelling.

Outline

- When Simulation Is the Appropriate tool
- When Simulation Is Not appropriate
- Advantages and Disadvantages of Simulation
- Areas of Application
- Systems and System Environment
- Components of a System
- Discrete and Continuous Systems
- Model of a System
- Types of Models
- Discrete-Event System Simulation
- Steps in a Simulation Study
- Monte Carlo Simulation algorithm
- Queuing Theory (Waiting Model)

ISIBOR O.O.
COMPUTER SCIENCE DEPARTMENT
SOUTHWESTERN UNIVERSITY
AND
LAGOS CITY POLYTECHNIC
IKEJA-NIGERIA.
osesisibor@gmail.com
08063546421.
FEBRUARY 2021

DEFINITION

A **Simulation** is the imitation of the operation of real-world process or system over time.

… Generation of artificial history and observation of that observation history

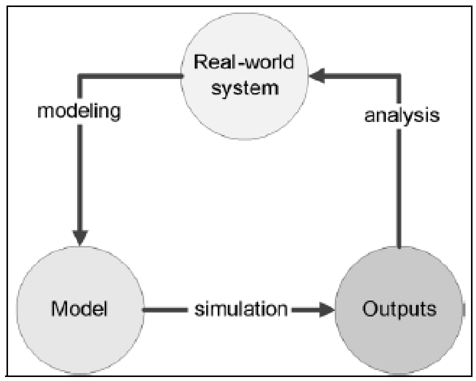
A **model** constructs a conceptual framework that describes a system

The behavior of a system that evolves over time is studied by developing a ***simulation model***.

„ The model takes a set of expressed assumptions:

… Mathematical, logical

… Symbolic relationship between the *entities*



Modeling, Simulation and Analysis

Goal of modeling and simulation

A **model** can be used to investigate a wide verity of "what if" questions about real-world system.

… Potential changes to the system can be simulated and predicate their impact on the system.

… Find adequate parameters before implementation

So, **simulation** can be used as

… Analysis tool for predicating the effect of changes

… Design tool to predicate the performance of new system

It is better to do simulation before Implementation.

How a model can be developed?

- Mathematical Methods
  … Probability theory, algebraic method,
  … Their results are accurate
  … They have a few Number of parameters
  … It is impossible for complex systems
- Numerical computer-based simulation
  … It is simple
  … It is useful for complex system

When Simulation Is the Appropriate Tool.,

1) Simulation enable the study of internal interaction of a subsystem with complex system
2) Informational, organizational and environmental changes can be simulated and find their effects
3) A simulation model helps us to gain knowledge about improvement of system
4) Finding important input parameters with changing simulation inputs
5) Simulation can be used with new design and policies before implementation
6) Simulating different capabilities for a machine can help determine the requirement

7) Simulation models designed for training make learning possible without the cost disruption
8) A plan can be visualized with animated simulation
9) The modern system (factory, wafer fabrication plant, service organization) is too complex that its internal interaction can be treated only by simulation

When Simulation Is Not appropriate.

1) When the problem can be solved by common sense.
2) When the problem can be solved analytically.
3) If it is easier to perform direct experiments.
4) If cost exceed savings.
5) If resource or time are not available.
6) If system behavior is too complex.
   - Like human behavior

Simulation technique is considered as a valuable tool because:

1) It has a wide area of application.
2) It can be used to solve and analyze large and complex real-world problems.
3) Simulation provides solutions to various problems in functional areas like production, marketing, finance, human resource, etc.
4) It is useful in policy decisions through corporate planning models.
5) Simulation experiments generate large amounts of data and information using a small sample data, which considerably reduces the amount of cost and time involved in the exercise.

Advantages of simulation and modeling

1) New policies, operating procedures, information flows and son on can be explored without disrupting ongoing operation of the real system.
2) New hardware designs, physical layouts, transportation systems and … can be tested without committing resources for their acquisition.
3) Time can be compressed or expanded to allow for a speed-up or slow-down of the phenomenon (clock is self-control).
4) Insight can be obtained about interaction of variables and important variables to the performance.
5) Bottleneck analysis can be performed to discover where work in process, the system is delayed.
6) A simulation study can help in understanding how the system operates.
7) “What if” questions can be answered.

Disadvantages of simulation and modeling

A. Model building requires special training.
   - Vendors of simulation software have been actively developing packages that contain models that only need input (templates).
B. Simulation results can be difficult to interpret.
C. Simulation modeling and analysis can be time consuming and expensive.
   - Many simulation software have output-analysis.

Areas of application

A. Manufacturing Applications
B. Semiconductor Manufacturing
C. Construction Engineering and project management
D. Military application

E. Logistics, Supply chain and distribution application
F. Transportation modes and Traffic
G. Business Process Simulation
H. Health Care
I. Automated Material Handling System (AMHS)
    - Test beds for functional testing of control-system software
J. Risk analysis
    - Insurance, portfolio,...
K. Computer Simulation
    - CPU, Memory,...
L. Network simulation
    - Internet backbone, LAN (Switch/Router), Wireless, PSTN (call center),...

Systems and System Environment

A *system* is defined as a group of objects that are joined together in some regular interaction toward the accomplishment of some purpose.

- An automobile factory: Machines, components parts and workers operate jointly along assembly line

A system is often affected by changes occurring outside the system: system environment.

- Factory : Arrival orders
    - Effect of supply on demand: relationship between factory output and arrival (activity of system)
- Banks: arrival of customers

A system, in general, is a collection of entities which are logically related and which are of interest to a particular application. **The following features of a system are of interest:**

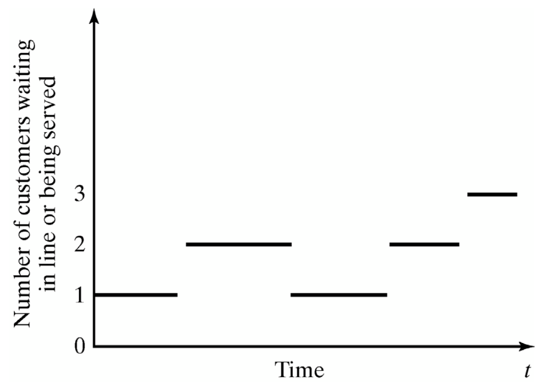
1. **Environment**: Each system can be seen as a subsystem of a broader system.
2. **Interdependency**: No activity takes place in total isolation.
3. **Sub-systems**: Each system can be broken down to sub-systems.
4. **Organization**: Virtually all systems consist of highly organized elements or components, which interact in order to carry out the function of the system.
5. **Change**: The present condition or state of the system usually varies over a long period of time.

Components of system

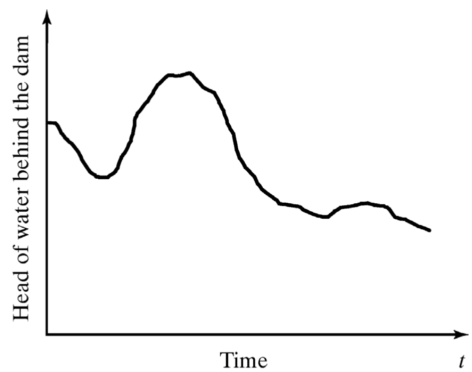
1) Entity: An object of interest in the system: Machines in factory
2) Attribute: The property of an entity: speed, capacity
3) Activity: A time period of specified length: welding, stamping
4) State: A collection of variables that describe the system in any time: status of machine (busy, idle, down)
5) Event: A instantaneous occurrence that might change the state of the system: breakdown
6) Endogenous: Activities and events occurring with the system
7) Exogenous: Activities and events occurring with the environment

Discrete and Continues Systems

A **discrete system** is one in which the state variables change only at a discrete set of points in time: Bank example



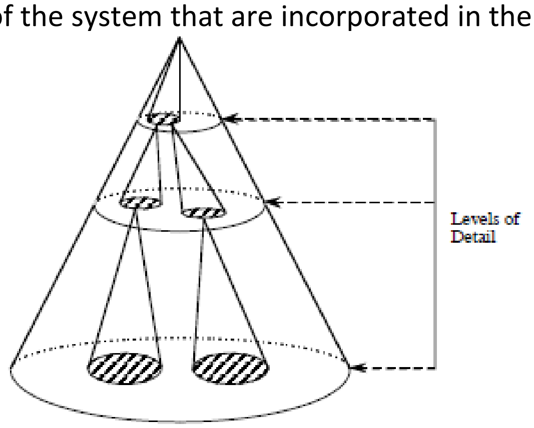
A continuous system is one in which the state variables change continuously over time: Head of water behind the dam



Model of a System

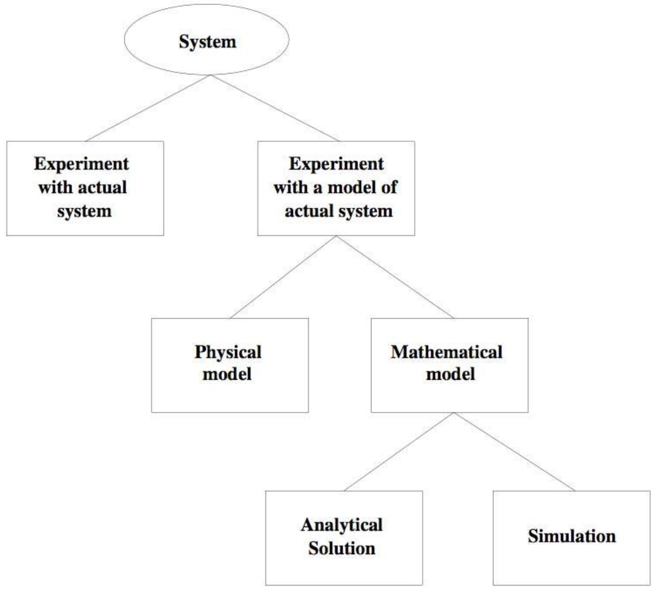
- To study the system
  - … it is sometimes possible to experiments with system
  - „ This is not always possible (bank, factory)
  - „ A new system may not yet exist
- Model: construct a conceptual framework that describes a system
  - … It is necessary to consider those accepts of systems that affect the problem under investigation (unnecessary details must remove)

When building a simulation model of a real-life system under investigation, one does not simulate the whole system. Rather, one simulates those sub-systems which are related to the problems at hand. This involves modelling parts of the system at various levels of detail. This can be graphically depicted using Beard's managerial pyramid as shown in the diagram below. The collection of blackened areas forms those parts of the system that are incorporated in the model.



BEARD'S managerial pyramid

Types of Models



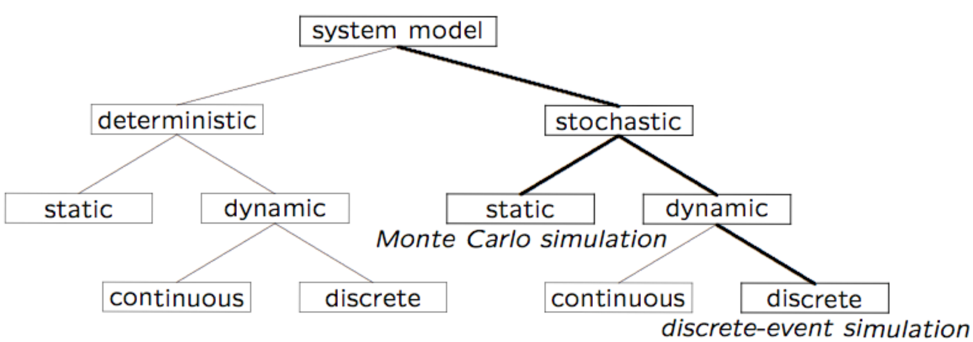
Characterizing a Simulation Model

- Deterministic or Stochastic
  - Does the model contain stochastic components?
  - Randomness is easy to add to a DES
- Static or Dynamic
  - Is time a significant variable?
- Continuous or Discrete
  - Does the system state evolve continuously or only at discrete points in time?
  - Continuous: classical mechanics
  - Discrete: queuing, inventory, machine shop models

Discrete-Event Simulation Model

1. Stochastic: some state variables are random
2. *Dynamic*: time evolution is important
3. *Discrete-Event*: significant changes occur at discrete time instances

Model Taxonomy



Discrete-Event Simulation (DES) Model Development

How to develop a model:

1. Determine the goals and objectives
2. Build a ***conceptual*** model
3. Convert into a ***specification*** model
4. Convert into a ***computational*** model
5. Verify
6. Validate

Typically, an iterative process.

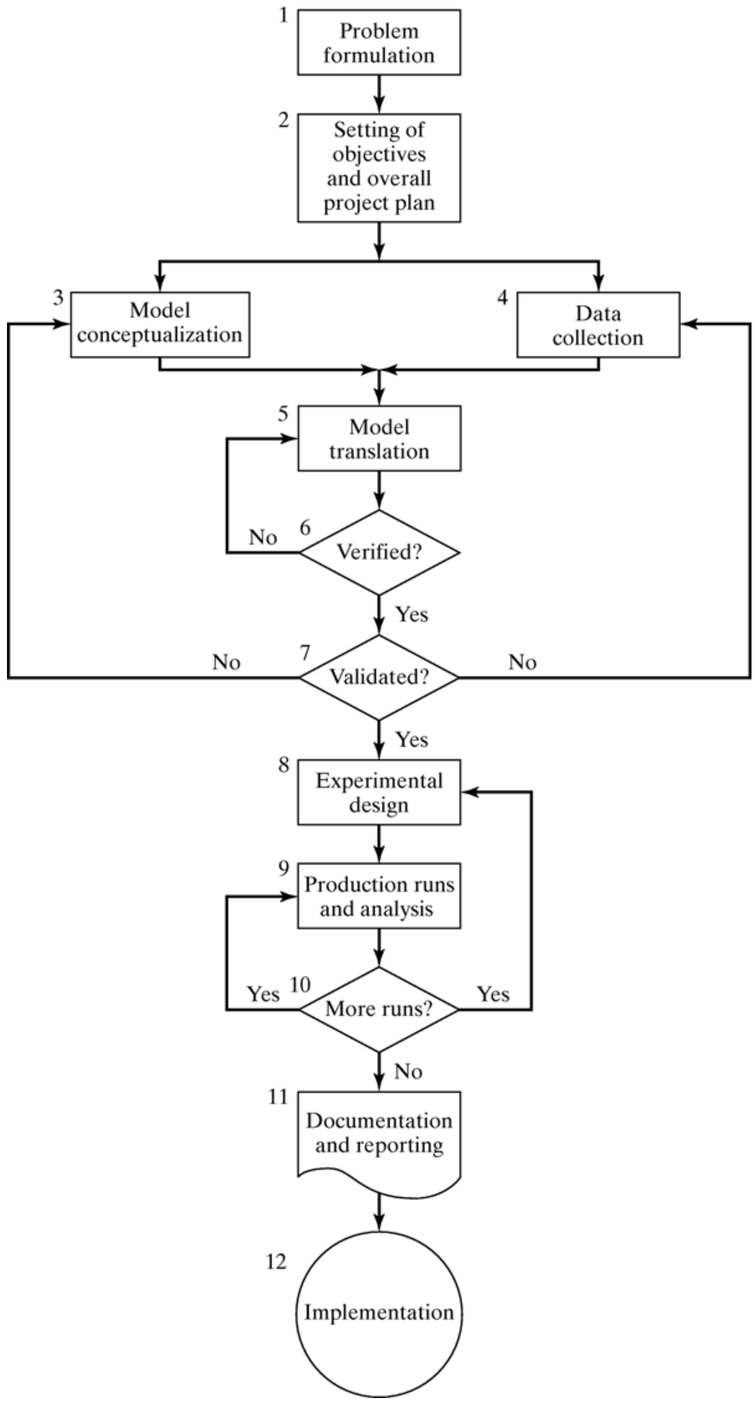
Three Model Levels

- Conceptual
    - … Very high level
    - … How comprehensive should the model be?
    - … What are the *state variables,* which are dynamic, and which are important?
- Specification
    - … On paper
    - … May involve equations, pseudocode, etc.
    - … How will the model receive input?
- Computational
    - … A computer program.
    - … General-purpose PL or simulation language?

Verification vs. Validation

- Verification
    - … Computational model should be consistent with specification model
    - … Did we build the <u>model right</u>?
- Validation
    - … Computational model should be consistent with the system being analyzed
    - … Did we build the <u>right model</u>?
    - … Can an expert distinguish simulation output from system output?
- Interactive graphics can prove valuable

Steps in Simulation Study



MONTE CARLO SIMULATION

In simulation, we have deterministic models and probabilistic models. Deterministic simulation models have the alternatives clearly known in advance and the choice is made by considering the various well-defined alternatives. Probabilistic simulation model is stochastic in nature and all decisions are made under uncertainty. One of the probabilistic simulation models is the Monte Carlo method. In this method, the decision variables are represented by a probabilistic distribution and random samples are drawn from probability distribution using random numbers. The simulation experiment is conducted until the required number of simulations are generated. Finally, the best course of action is selected for implementation. The significance of Monte Carlo Simulation is that decision variables may not explicitly follow any standard probability distribution such as Normal, Poisson, Exponential, etc. The distribution can be obtained by direct observation or from past records.

ALGORITHM/PROCEDURE FOR MONTE CARLO SIMULATION:

Step 1: Establish a probability distribution for the variables to be analyzed.
Step 2: Find the cumulative probability distribution for each variable.
Step 3: Set Random Number intervals for variables and generate random numbers.
Step 4: Simulate the experiment by selecting random numbers from random numbers tables until the required number of simulations are generated.
Step 5: Examine the results and validate the model.

SIMULATION OF DEMAND FORECASTING PROBLEM

QUESTION 1: An ice-cream parlor's record of previous month's sale of a particular variety of ice cream as follows.

Simulation of Demand Problem

Demand (No. of Ice-creams)	No. of days	Probability
4	5	0.17
5	10	0.33
6	6	0.20
7	8	0.27
8	1	0.03

Simulate the demand for first 10 days of the month using the following random numbers:
17, 46, 85, 09, 50, 58, 04, 77, 69 and 74

Demand	Probability	Cumulative Probability	Random Number Interval
4	0.17	0.17	00-16
5	0.33	0.50	17-49
6	0.20	0.70	50-69
7	0.27	0.97	70-96
8	0.03	1.00	97-99

To simulate the demand for ten days, select ten random numbers from random number tables. The random numbers selected are, 17, 46, 85, 09, 50, 58, 04, 77, 69 and 74
The first random number selected, 7 lies between the random number interval 17-49 corresponding to a demand of 5 ice-creams per day. Hence, the demand for day one is 5. Similarly, the demand for the remaining days is simulated as shown in Table 15.4.

Demand Simulation

Day	1	2	3	4	5	6	7	8	9	10-
Random Number	17	46	85	9	50	58	4	77	69	74
Demand	5	5	7	4	6	6	4	7	6	7

- **Average demand** = summation of simulated demand / No of simulated days
  57/10 = 5.7 ice cream per day.
- **The expected demand /day** can be computed as, $\sum_{i=0}^{n} P_i X_i$
  Expected demand per day
  where, **pi** = probability and **xi** = demand
  = (4 * 0.17) + (5 * 0.33) + (6 * 0.20) + (7 * 0.27) + (8 [illegible]
  = 5.66 ice cream per day

QUESTION 3: A dealer sells a particular model of washing machine for which the probability distribution of daily demand is as give below:

Probability Distribution of Daily Demand

Demand/day	0	1	2	3	4	5
Demand	0.05	0.25	0.20	0.25	0.10	0.15

i. Ten random numbers that have been selected from random number tables are 68, 47, 92, 76, 86, 46, 16, 28, 35, 54, find the demand for ten days.
ii. Find the Average Demand of washing machines per day.
iii. Find the Expected Demand of washing machines per day.

Solution: Assign sets of two-digit random numbers to demand levels as shown in the table below

Random Numbers Assigned to Demand

Demand	Probability	Cumulative Probability	Random Number Intervals
0	0.05	0.05	00-04
1	0.25	0.30	05-29
2	0.20	0.50	30-49
3	0.25	0.75	50-74
4	0.10	0.85	75-84
5	0.15	1.00	85-99

Ten Random Numbers Selected

Trial No	Random Number	Demand / day
1	68	3
2	47	2
3	92	5
4	76	4
5	86	5
6	46	2
7	16	1
8	28	1
9	35	2
10	54	3
Total Demand		28

- **Average demand** = summation of simulated demand / No of simulated days
  28/10 = 2.8 washing machines per day.
- **The expected demand /day** can be computed as, $\sum_{i=0}^{n} P_i X_i$
  Expected demand per day
  where, **pi** = probability and **xi** = demand
  = (0.05 × 0) + (0.25 × 1) + (0.20 × 2) + (0.25 × 3) + (0.1 × 4) + (0.15 × 5)
  = 2.55 washing machines.

The average demand of 2.8 washing machines using ten-day simulation differs significantly when compared to the expected daily demand. If the simulation is repeated number of times, the answer would get closer to the expected daily demand.

QUESTION 3: A farmer has 10 acres of agricultural land and is cultivating tomatoes on the entire land. Due to fluctuation in water availability, the yield per acre differs. The probability distribution yields are given below:

a. The farmer is interested to know the yield for the next 12 months if the same water availability exists. Simulate the average yield using the following random numbers 50, 28, 68, 36, 90, 62, 27, 50, 18, 36, 61 and 21, given in Table below:
b. Find the Average Demand per day.
c. Find the Expected Demand per day.

Yield of tomatoes per acre (kg)	Probability
200	0.15
220	0.25
240	0.35
260	0.13
280	0.12

WAITING MODEL (QUEUING THEORY)

Queuing theory deals with problems that involve waiting (or queuing). Examples of queues or long waiting lines that occurs every day in our daily life might be:

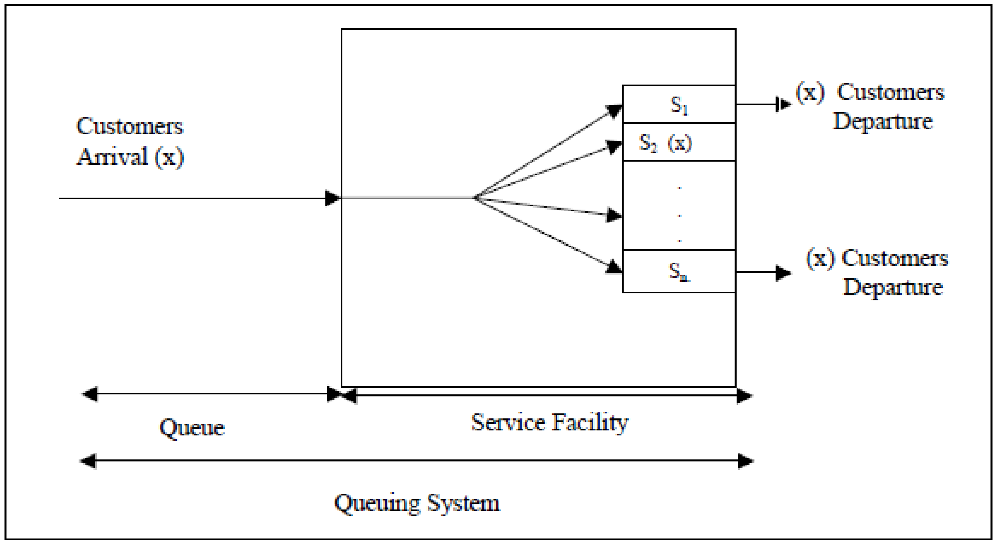
- Waiting for checking out at the Supermarket.
- Waiting for service in banks and at reservation counters.
- Waiting at the telephone booth or a barber's saloon.
- Waiting for a train or a bus.
- Waiting for a haircut
- Ticket-booking counters

Whenever a customer arrives at a **service facility**, some of them usually have to wait before they receive the desired service. This forms a **queue or waiting line** and **customers** feel discomfort either mentally or physically because of long waiting queue.
Queuing theory explores and measures the performance in a queuing situation such as average number of customers waiting in the queue, average waiting time of a customer and average server utilization.

QUEUING SYSTEMS
In general, a queuing system comprises with two components,

1) **The queue**: This is where the customers are waiting to be served
2) **The service facility**: This is customers being served and the individual service stations

A general queuing system with parallel server is shown below:



A typical queuing system

CHARACTERISTICS OF QUEUING SYSTEM

In designing a good queuing system, it is necessary to have a good information about the model. The characteristics listed below would provide sufficient information.

1) **The arrival pattern**: The arrival pattern describes how a customer may become a part of the queuing system. The arrival time for any customer is unpredictable. Therefore, the arrival time and the number of customers arriving at any specified time intervals are usually random variables.

2) **The service mechanism**: The service mechanism is a description of resources required for service. If there are infinite number of servers, then there will be no queue. If the number of servers is finite, then the customers are served according to a specific order. The time taken to serve a particular customer is called the service time.

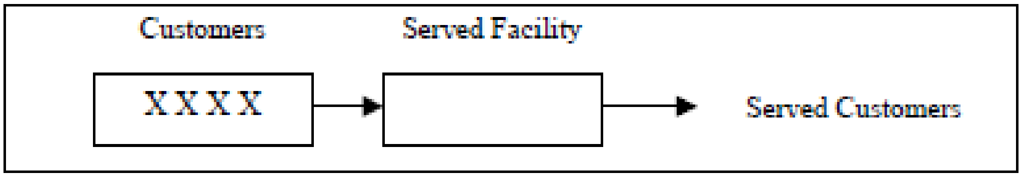
3) **The queue discipline**: The most common queue discipline is the **"First Come First Served" (FCFS)** or **"First-in, First-out" (FIFO)**. Situations like waiting for a haircut, ticket-booking counters follow FCFS discipline. Other disciplines include **"Last In First Out" (LIFO)** where last customer is serviced first, **"Service In Random Order" (SIRO)** in which the customers are serviced randomly irrespective of their arrivals.

4) **The number of customers allowed in the system:** Some of the queuing processes allow the limitation to the capacity or size of the waiting room, so that the waiting line reaches a certain length, no additional customers is allowed to enter until space becomes available by a service completion.
5) **The number of service channels**: The more the number of service channels in the service facility, the greater the overall service rate of the facility. The combination of arrival rate and service rate is critical for determining the number of service channels.

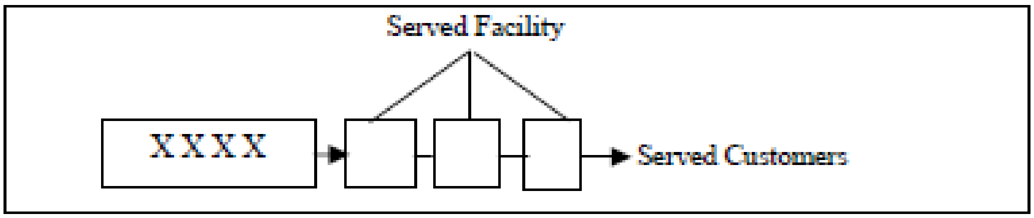
ARRANGEMENT OF SERVICE FACILITIES

a. **Arrangement of service facilities in series**: This can come in two different forms as shown below:

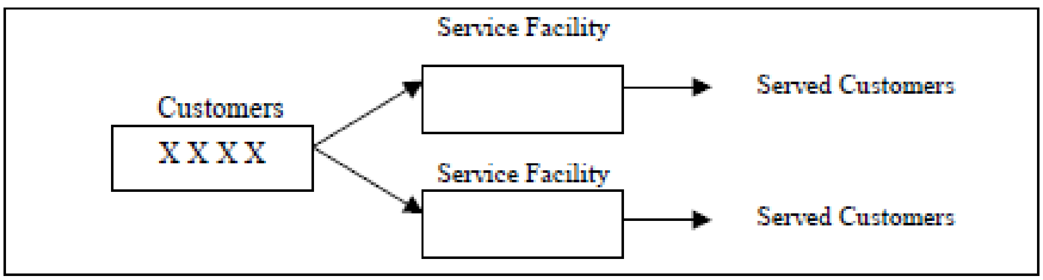
   i. **Single Queue Single Server**



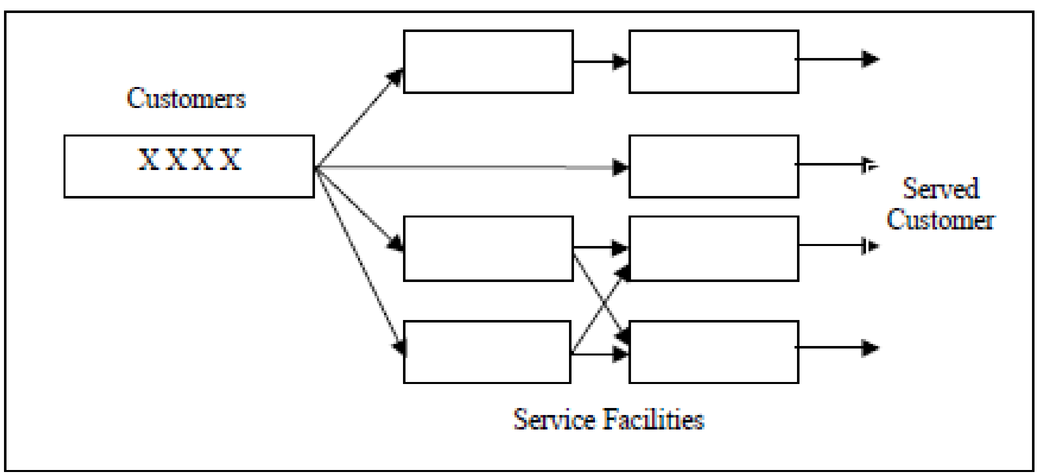
   ii. **Single Queue, Multiple Server**



b. **Arrangement of Service facilities in Parallel**



c. **Arrangement of Mixed Service facilities**



Attitude of Customers

- **Patient Customer:** Customer arrives at the service system, stays in the queue until served, no matter how much he has to wait for service.
- **Impatient Customer:** Customer arrives at the service system, waits for a certain time in the queue and leaves the system without getting service due to some reasons like long queue before him.
- **Balking:** Customer decides not to join the queue by seeing the number of customers already in service system.
- **Reneging:** Customer after joining the queue, waits for some time and leaves the service system due to delay in service.
- **Jockeying:** Customer moves from one queue to another thinking that he will get served faster by doing so.

To analyze queuing situations, the questions of interest that are typically concerned with measures of queuing system performance include (Questions of interest that will arise from analyzing the performance of a queuing system),

1. What will be the waiting time for a customer before service is complete?
2. What will be the average length of the queue?
3. What will be the probability that the queue length exceeds a certain length?
4. How can a system be designed at minimum total cost?
5. How many servers should be employed?
6. Should priorities of the customers be considered?
7. Is there sufficient waiting area for the customers?

SYMBOLS AND NOTATIONS

The symbols and notations used in queuing system are as follows:

n = Number of customers in the system (both waiting and in service).

λ (Lamda) = Average number of customers arriving per unit of time.

μ = Average number of customers being served per unit of time.

λ / μ = P, traffic intensity.

C = Number of parallel service channels (i,e., servers).

L_s = Average or expected number of customers in the system (both waiting and in service).

L_q = Average or expected number of customers in the queue.

W_s = Average waiting time in the system (both waiting and in service).

W_q = Average waiting time of a customer in the queue.

P_n = Time independent probability that there are n customers in the system (both waiting and in service).

P_n (t) = Probability that there are n customers in the system at any time t (both waiting and in service).

QUEUING EQUATIONS

λ (Lamda) = (Arrival Rate) Average number of customers arriving per unit of time. μ = (Service Rate) Average number of customers being served per unit of time.		
	SYSTEM	**QUEUE**
LENGTH (NUMBER)	$L_s = \frac{\lambda}{\mu - \lambda}$	$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$
WAITING TIME	$W_s = \frac{1}{\mu - \lambda}$	$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$
$P_0 = 1 - \frac{\lambda}{\mu}$		
Probability that there will be customers waiting in the system = 1 - P_0		
Traffic intensity = $\frac{\lambda}{\mu}$		

Example 1

Consider a situation where the mean arrival rate (l) is one customer every 4 minutes and the mean service time (m) is 2½ minutes. Calculate the average number of customers in the system, the average queue length and the time taken by a customer in the system and the average time a customer waits before being served. Also, calculate the probability that there will be customers waiting in the system. Also, Calculate the probability that there will be customers waiting in the system and the probability that there are no customers waiting in the system.

Solution:

Given, Average Arrival Rate **λ** = 1 customer every 4 minutes or 15 customers per hour

Average Service -Rate **μ** = 1 customer every 2½ minutes or 24 customers per hour

(i) The average number of customers in the system, $L_S = \frac{\lambda}{\mu - \lambda} = \frac{15}{24 - 15}$ **= 1.66 customers**

(ii) The average queue length, $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{15^2}{24(24 - 15)}$ **= 1.04 customers**

(iii) The average time a customer spends in the system, $W_S = \frac{1}{\mu - \lambda} = \frac{1}{24 - 15}$ **= 0.11 Seconds**

0.11s × 60 = 6.66 minutes

(iv) The average time a customer waits before being served, $W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{15}{24(24-15)} =$ **0.0069 Seconds** = 0.069s × 60 = 4.16 minutes

(v.) $P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{15}{24} = 0.58$

(vi.) **Probability that there will be customers waiting in the system = 1 - P_0 = 1 – 0.58 = 0.42**

Example 2

Auto car service provides a single channel water wash service. The incoming arrivals occur at the rate of 4 cars per hour and the mean service rate is 8 cars per hour. Assume that arrivals follow a Poisson distribution and the service rate follows an exponential probability distribution. Determine the following measures of performance:

(a.) What is the average number of cars in the system?
(b.) What is the average length of cars in the queue?
(c.) What is the average time a car spends in the system?
(d.) What is the average time that a car waits for water – wash to begin?
(e.) Calculate the probability that there will be customers waiting in the system.
(f.) Calculate the probability that there are no customers waiting in the system.

Solution:
Given, Average Arrival Rate **λ** = 12 trucks per day
Average Service -Rate **μ** = 18 trucks per day

(i.) The average number of customers in the system, $L_S = \frac{\lambda}{\mu - \lambda} = \frac{12}{18-12}$ **= 2 trucks**

(ii.) The average queue length, $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{12^2}{18(18-12)}$ **= 1.33 Trucks**

(iii.) The average time a customer spends in the system, $W_S = \frac{1}{\mu - \lambda} = \frac{1}{18-12}$ **= 0.16667 Seconds**

0.16667s × 60 = 10 minutes

(iv.) The average time a customer waits before being served, $W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{12}{18(18-12)} =$ **0.1111 Seconds = 0.1111s × 60 = 53.3 minutes**

(v.) $P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{12}{18} = 0.3333$

(vi.) **Probability that there will be customers waiting in the system = 1 - P_0 = 1 – 0.3333 = 0.66667**